

The Equivalence Theorem and Effective Lagrangians

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Abstract

We point out that the equivalence theorem, which relates the amplitude for a process with external longitudinally polarized vector bosons to the amplitude in which the longitudinal vector bosons are replaced by the corresponding pseudo-Goldstone bosons, is not valid for effective Lagrangians. However, a more general formulation of this theorem also holds for effective interactions. The generalized theorem can be utilized to determine the high-energy behaviour of scattering processes just by power counting and to simplify the calculation of the corresponding amplitudes. We apply this method to the phenomenologically most interesting terms describing effective interactions of the electroweak vector and Higgs bosons in order to examine their effects on vector-boson scattering and on vector-boson-pair production in $f\bar{f}$ annihilation. The use of the equivalence theorem in the literature is examined.

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1 Introduction

The equivalence theorem (ET) [1, 2, 3, 4] has become a useful tool for investigating the high-energy behaviour of scattering processes with external longitudinally polarized vector bosons, especially vector-boson scattering, $V_L V_L \rightarrow V_L V_L$, and vector-boson-pair production in fermion-antifermion annihilation, $f\bar{f} \rightarrow V_L V_L$. This theorem states that the amplitude for such a process is equal to the amplitude (calculated within the R_ξ -gauge) in which the external longitudinal vector bosons are replaced by the corresponding pseudo-Goldstone bosons (times a phase factor), apart from corrections which are of order M/E with M being the vector-boson mass and E being its energy¹:

$$\begin{aligned} \mathcal{M}(V_{L,1} \dots V_{L,N_1} A \rightarrow V_{L,1} \dots V_{L,N_2} B) \\ = i^{N_1} (-i)^{N_2} \mathcal{M}(\phi_1 \dots \phi_{N_1} A \rightarrow \phi_1 \dots \phi_{N_2} B) + O\left(\frac{M}{E}\right) \end{aligned} \quad (1.1)$$

($V_{L,i}$, ϕ_i , A and B denote the longitudinal vector bosons, the corresponding Goldstone bosons and all other in- and outgoing particles, respectively.) The ET was first proven (within the electroweak standard model (SM)) in [2]. A simpler proof was given in [3].

The ET simplifies calculations of S -matrix elements, because on the r.h.s. of (1.1) no gauge cancellations occur. E.g., if one wants to calculate the amplitude for a process $V_L V_L \rightarrow V_L V_L$ within the SM without using the ET, one has to consider (in the tree approximation) a diagram with a four-vector contact interaction, vector exchange diagrams and scalar exchange diagrams. The single Feynman diagrams grow with increasing total energy E as E^4 (contact and vector-exchange) or E^2 (scalar exchange) for $E \gg M_H$. However, when summing up the diagrams all terms which grow with positive powers of E cancel and the resulting amplitude is proportional to E^0 . Thus, to calculate the $O(E^0)$ -part of the amplitude one also has to consider the nonleading contributions of the single diagrams. If one applies instead the ET one has to calculate $\phi\phi \rightarrow \phi\phi$, viz. contact, Higgs-exchange and vector-exchange diagrams with four external Goldstone bosons. All these diagrams are at most $O(E^0)$ and thus no cancellations occur. Therefore it is sufficient to consider only the leading part of the single diagrams.

During the last few years, vector-boson scattering and vector-boson-pair production have been investigated within effective electroweak theories which contain nonstandard interactions among the vector bosons and nonstandard interactions between vectors and scalars in order to investigate the effects of these theories on future experiments. When considering an effective Lagrangian one assumes that there exists an (unknown) renormalizable theory (“new physics”) which involves in addition to the known particles also heavy particles. At an energy scale much lower than the mass of the heavy particles these can be removed from the theory by expressing their effects through effective (nonrenormalizable) interactions of the light particles. Two scenarios how such an effective theory can be generated have been investigated in the literature:

1. If there is a relatively light Higgs boson, the effective Lagrangian consists of the SM Lagrangian plus additional effective interaction terms of higher dimension which are also gauge invariant with respect to the electroweak gauge group $SU(2) \times U(1)$ and in which the scalar sector is linearly realized like in the SM Lagrangian [5].

¹This means that amplitudes which decrease with increasing energy E cannot be determined by applying the ET.

2. On the other hand one can assume that the Higgs boson is heavy or even does not exist, i.e. the symmetry breaking sector is (at least a part of) the new physics. In this case one adds effective interaction terms to the Lagrangian of the gauged nonlinear σ -model (GNLSM). The GNLSM is the limit of the SM for infinite Higgs Mass [6]. In this (nonrenormalizable) gauge theory the scalar sector is nonlinearly realized. The additional effective terms are also $SU(2) \times U(1)$ gauge invariant with the scalar sector being nonlinearly realized. They can be generated by the heavy Higgs sector through loop effects [6] or by other effects of the new physics.

In order to calculate S -matrix elements within such an effective theory, one could be tempted to use the ET. In general, however, this is not correct because *the equivalence theorem is not valid within effective theories*; the proof of the ET within the SM given in [2, 3, 4] cannot be generalized to this case. This can easily be seen as follows: The proof consists of two steps:

1. First, the following identity for S -matrix elements with external longitudinal vector bosons is derived:

$$\begin{aligned} & \mathcal{M}(V_{L,1} \dots V_{L,N_1} A \rightarrow V_{L,1} \dots V_{L,N_2} B) \\ &= \sum_{M_1=0}^{N_1} \sum_{M_2=0}^{N_2} i^{M_1} (-i)^{M_2} [\mathcal{M}(\phi_1 \dots \phi_{M_1} v_{M_1+1} \dots v_{N_1} A \rightarrow \phi_1 \dots \phi_{M_2} v_{M_2+1} \dots v_{N_2} B) \\ & \quad + \text{permutations of the } \phi \text{ s and } v \text{ s}]. \end{aligned} \quad (1.2)$$

v stands for an external vector boson with its longitudinal polarization vector being substituted by the nonleading part

$$v^\mu = \epsilon_L^\mu - \frac{P^\mu}{M} \quad (1.3)$$

(with P^μ being the four-momentum of the particle) which is $O(M/E)$. We will refer to the relation (1.2) as the generalized equivalence theorem (GET). The GET expresses an S -matrix element as a sum of all amplitudes that can be constructed by replacing each longitudinal vector boson either by a Goldstone boson or its polarization vector ϵ_L by the nonleading term v (1.3) (multiplied by appropriate phase factors). In distinction to the ET (1.1), which only determines the $O(E^0)$ -terms, the GET (1.2) is a correct relation, in which no approximation is made.

2. In the second step, (1.1) is proven by showing that all terms on the r.h.s. of (1.2) except for the one with $M_1 = N_1$, $M_2 = N_2$ are $O(M/E)$ within the SM.

The proof of the GET (1.2) is only based on the BRS invariance and thus on the gauge invariance of the quantized Lagrangian [2, 3, 4]. It is not affected by adding effective interaction terms to the theory. This means that the GET is not only valid for the SM but also for any effective gauge theory². However, the derivation of the ET (1.1) from (1.2) requires that all amplitudes calculated within the theory do not increase with increasing energy, i.e. that they behave at most as $O(E^0)$ [2, 4]. Therefore, the ET is not a consequence

²In [2] this was shown only for theories with a linearly realized scalar sector. However the alternative proof in [3] holds for both, linear and nonlinear models. Recently, a proof analogous to the one in [2] has been done for the nonlinear case [7].

of BRS invariance (and thus gauge invariance) alone, it follows from BRS invariance *and* good high-energy behaviour. The latter requirement is not fulfilled within effective theories³, in which amplitudes in general increase with increasing energy due to the effects of the additional interaction terms.

Applying this reasoning to the SM, this means that a priori the ET only holds for $E \gg M_H$, because only in this case the amplitudes behave at most as $O(E^0)$. At energies lower than the Higgs mass the amplitudes increase with E [1]. However, it has been shown in [4] that the ET is also valid within the SM for $E \ll M_H$. This means that the ET even holds within the GNLSM, because this is the heavy-Higgs limit of the SM.

Furthermore we want to remind the reader of the fact that the ET only states that *external* longitudinal vector bosons can be replaced by Goldstone bosons; *internal* vector lines still have to be considered. Thus, in general, *the interactions of longitudinally polarized vector bosons cannot be described by a Lagrangian with only scalar fields* in contradiction to the original formulation of the ET in [1]. For example, if one calculates $\phi\phi \rightarrow \phi\phi$ in order to determine the $O(E^0)$ terms of the amplitude for $V_L V_L \rightarrow V_L V_L$ within the SM, the vector-exchange diagrams cannot be neglected if $E \gg M_H$.

In this article we will show that the GET (1.2), which occurs as a byproduct of the proof of the ET within the SM and which also holds for effective Lagrangians, is a very useful tool to determine the high-energy behaviour of scattering processes calculated within effective theories. Our analysis will be carried out within tree approximation⁴ which is no severe restriction since, due to the smallness of the effective terms, phenomenological investigations of such terms are usually carried out at the tree level. The GET (1.2) expresses the amplitude for a process with longitudinal vector bosons by a sum of amplitudes with external ϕ s and *vs.* Like in the SM, on the r.h.s. of (1.2) in general no cancellations occur, i.e. the single Feynman diagrams that contribute to the r.h.s. have the high-energy behaviour that they are supposed to have by power counting and the leading terms do not cancel when summing up the various diagrams. This means that in order to calculate the leading term of an S -matrix element, one can determine by simple power counting which of the diagrams on the r.h.s. contribute to the highest powers of E . Then one only has to calculate the leading contributions to these diagrams. The only difference to the SM is that, in general, not just those diagrams with all external V_L s replaced by ϕ s contribute to the leading terms, there may be contributions from some diagrams with *vs.*

Such a power-counting method cannot be applied if the amplitudes are calculated directly without using the GET, because even in effective theories cancellations occur [10, 11, 12]. For example, if one adds to the SM Lagrangian a dimension-six quadrupole interaction term, the additional contributions to the single Feynman diagrams for $V_L V_L \rightarrow V_L V_L$ are by power counting supposed to diverge as E^6 but due to cancellations they are $O(E^2)$ and when summing them up the resulting amplitude turns out to be only $O(E^0)$ [11, 12]. We will show below that this is obvious if one uses the GET, because all diagrams on the r.h.s. of (1.2) are at most $O(E^0)$ within this special effective theory. Furthermore we will see that this quadrupole interaction is an example for the invalidity of the ET within effective theories, because the quadrupole term yields $O(E^0)$ corrections to the SM

³In a recent work [8] it has been claimed that the ET also holds for effective Lagrangians with additional nonstandard interaction terms. This is not true; the authors of [8] only observe that the proof of (1.2) also holds for effective Lagrangians and they erroneously conclude that this implies the ET (1.1).

⁴It should be noted that one has to consider correction factors stemming from the renormalization of the external lines if one applies the ET beyond the tree-level [9].

amplitude, while the ET (1.1) would predict that even those are absent since this term yields no additional interactions involving scalar fields. The $O(E^0)$ corrections, however can easily be calculated by using the GET. Other dimension-six interaction terms in general yield an $O(E^2)$ behaviour of the amplitudes, although the single diagrams with external longitudinal vectors behave as $O(E^4)$ [12]. This can also easily be seen from the GET.

In this article we will apply the power-counting method based on the GET (1.2) to the phenomenologically most interesting terms that describe effective interactions of the electroweak vector and Higgs bosons. We will consider both linear models with a Higgs boson and nonlinear models without a Higgs boson. We will analyze the effects of these terms on the high-energy behaviour of the processes $VV \rightarrow VV$ and $f\bar{f} \rightarrow VV$, where the external V s may be longitudinal or transversal. We will find that our results agree with those obtained by a direct calculation [12].

Although the ET (1.1) is not valid for effective Lagrangians, it was applied in several articles in order to calculate S -matrix elements within effective theories [13, 14, 15]. In [13], effective interactions of longitudinal vector bosons were even described by a Lagrangian with only scalar fields, i.e. even internal vector lines were neglected. On the basis of our results we will critically analyze these works. We will find that indeed in most of these articles the high-energy effects of several effective interaction terms are not correctly determined, however that the numerical size of the errors is in general small.

This article is organized as follows: In Section 2 we introduce the effective interaction terms that we will consider. We examine both linear effective Lagrangians with a Higgs boson and nonlinear ones without. The validity of the generalized equivalence theorem (1.2) within effective theories is discussed. We determine which vertices follow from the effective interaction terms and the powers of E on that they depend. In Section 3 we apply the power-counting method based on the GET (1.2) to these effective Lagrangians in order to examine their effects on the processes $VV \rightarrow VV$ and $f\bar{f} \rightarrow VV$. We show that the high-energy behaviour can easily be determined and that the calculations become simplified. The results of this analysis are compared with those obtained by direct calculations. Phenomenological consequences are discussed. In Section 4 we critically analyze those articles in which the equivalence theorem has been applied to effective theories. Section 5 contains some concluding remarks.

2 The Effective Interaction Terms

In this section we introduce the effective interaction terms that will be examined later in this article. We consider both linear and nonlinear Lagrangians. We restrict to P , C and CP invariant nonstandard interactions. We discuss these terms only briefly, because this has been done in more detail elsewhere [6, 12, 16, 17, 18].

We use the following notation:

$$W_\mu = \frac{1}{2}W_{\mu i}\tau_i, \quad (2.1a)$$

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu], \quad (2.1b)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (2.1c)$$

$$\Phi = \frac{1}{\sqrt{2}}((v+H)\mathbf{1} + i\phi_i\tau_i), \quad (2.1d)$$

$$D_\mu \Phi = \partial_\mu \Phi + igW_\mu \Phi - \frac{i}{2}g'\Phi\tau_3 B_\mu. \quad (2.1e)$$

$W_{\mu i}$ and B_μ denote the gauge fields, g and g' the coupling constants, v the vacuum expectation value, H the Higgs field, ϕ_i the pseudo-Goldstone fields and τ_i the Pauli-matrices.

First we consider the case that a Higgs boson exists (with a mass which is light in comparison to the energy of the scattering process under consideration). In this case the effects of new physics can be parametrized by adding gauge invariant terms of higher (mass) dimension to the SM Lagrangian [5]. I.e. one considers an effective Lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_{n>4} \sum_i \frac{\epsilon_i}{\Lambda^{n-4}} \mathcal{L}_i^{(n)}, \quad (2.2)$$

where \mathcal{L}_{SM} is the (dimension-four) SM Lagrangian

$$\begin{aligned} \mathcal{L}_{SM} = & -\frac{1}{2} \text{tr} (W_{\mu\nu} W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + \frac{1}{2} \text{tr} [(D_\mu \Phi)^\dagger (D^\mu \Phi)] - \frac{1}{2} \mu^2 \text{tr} (\Phi^\dagger \Phi) - \frac{1}{4} \lambda [\text{tr} (\Phi^\dagger \Phi)]^2 \\ & + \text{fermionic terms}, \end{aligned} \quad (2.3)$$

$\mathcal{L}_i^{(n)}$ are the effective interaction terms of dimension n , ϵ_i are effective coupling constants and Λ is the scale of the new physics.

The phenomenologically most important effective terms are those of dimension six, since terms of higher dimension are suppressed by higher negative powers of Λ . The complete list of dimension-six terms is given in [5]. Most of these terms contain fermionic couplings or affect the gauge-boson propagators. This would yield tree-level effects on the processes $f\bar{f} \rightarrow f\bar{f}$ that have been measured at LEP I and other present experiments. Since no deviations from the SM have been found (within the experimental accuracy), we know that the coupling constants of these terms are very small [16, 18]. Therefore we restrict ourselves to those effective terms, which contain only effective interactions among the vector bosons and vector-scalar interactions and thus have no tree-level effects on $f\bar{f} \rightarrow f\bar{f}$ but on the processes $f\bar{f} \rightarrow VV$ and $VV \rightarrow VV$, which will be investigated in future experiments like LEP II, NLC ($e^+e^- \rightarrow W^+W^-$) or LHC ($q\bar{q} \rightarrow VV$ and $VV \rightarrow VV$ as subprocesses of $pp \rightarrow VVX$). These terms are [16, 18]:

$$\mathcal{L}_W = -\frac{2}{3}i \text{tr} (W_\mu{}^\nu W_\nu{}^\lambda W_\lambda{}^\mu), \quad (2.4a)$$

$$\mathcal{L}_{W\Phi} = i \text{tr} [(D_\mu \Phi)^\dagger W^{\mu\nu} (D_\nu \Phi)], \quad (2.4b)$$

$$\mathcal{L}_{B\Phi} = -\frac{1}{2}i \text{tr} [\tau_3 (D_\mu \Phi)^\dagger (D_\nu \Phi)] B^{\mu\nu}, \quad (2.4c)$$

$$\mathcal{L}_{WW} = -\frac{1}{8} \text{tr} (\Phi^\dagger \Phi) \text{tr} (W_{\mu\nu} W^{\mu\nu}), \quad (2.4d)$$

$$\mathcal{L}_{BB} = -\frac{1}{16} \text{tr} (\Phi^\dagger \Phi) B_{\mu\nu} B^{\mu\nu}, \quad (2.4e)$$

$$\mathcal{L}_D = \frac{1}{8} \text{tr} (\Phi^\dagger \Phi) \text{tr} [(D_\mu \Phi)^\dagger (D^\mu \Phi)]. \quad (2.4f)$$

The term \mathcal{L}_W (quadrupole term) contains nonstandard vector-boson self-interactions but no interactions with scalar fields. $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$ contain vector-boson self-interactions and

vector–scalar interactions. \mathcal{L}_{WW} , \mathcal{L}_{BB} and \mathcal{L}_D contain terms quadratic in the gauge fields, and thus it seems as if they affect the vector-boson propagators. However, after a redefinition of the fields and coupling constants all expressions that contain only gauge fields can be absorbed into the SM Lagrangian [16]. Thus these terms effectively only parametrize nonstandard vector–scalar interactions.

In addition to these dimension-six terms we also consider the following dimension eight-terms:

$$\mathcal{L}'_W = -\frac{1}{2}i \operatorname{tr} (\tau_3 \Phi^\dagger W_\mu{}^\nu W_\nu{}^\lambda \Phi) B_\lambda{}^\mu, \quad (2.5a)$$

$$\mathcal{L}'_{W\Phi} = \frac{1}{4}i \operatorname{tr} [\tau_3 (D_\mu \Phi)^\dagger (D_\nu \Phi)] \operatorname{tr} (\tau_3 \Phi^\dagger W^{\mu\nu} \Phi), \quad (2.5b)$$

$$\mathcal{L}'_{DD1} = -\frac{1}{4} \operatorname{tr} [(D_\mu \Phi)^\dagger (D_\nu \Phi)] \operatorname{tr} [(D^\mu \Phi)^\dagger (D^\nu \Phi)], \quad (2.5c)$$

$$\mathcal{L}'_{DD2} = -\frac{1}{4} \operatorname{tr} [(D_\mu \Phi)^\dagger (D^\mu \Phi)] \operatorname{tr} [(D_\nu \Phi)^\dagger (D^\nu \Phi)]. \quad (2.5d)$$

All these terms contain vector-boson self-interactions and vector–scalar interactions. They are the phenomenologically most interesting dimension-eight terms for the following reasons: \mathcal{L}'_W and $\mathcal{L}'_{W\Phi}$ yield together with the dimension-six terms \mathcal{L}_W , $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$ the most general, C , P , CP and locally $U(1)_{\text{em}}$ invariant trilinear vector-boson self-interactions [17]. \mathcal{L}'_{DD1} and \mathcal{L}'_{DD2} contain quadrilinear interactions among the gauge fields but no trilinear ones [6, 15, 18].

Next we consider the case that the Higgs boson is very heavy or even no Higgs boson exists. Then the effects of new physics should be parametrized by taking the limit $M_H \rightarrow \infty$ of an effective Lagrangian of type (2.2). This limit is obtained by substituting the linearly realized scalar fields in Φ (2.1d) through a nonlinear expression which contains only Goldstone bosons but no Higgs boson [6, 19]:

$$\Phi \rightarrow \frac{v}{\sqrt{2}} U \quad (2.6)$$

with

$$U = \exp \left(\frac{i\phi_i \tau_i}{v} \right). \quad (2.7)$$

Applying (2.6) to the SM (2.3) one obtains the gauged nonlinear σ model [6] with the Lagrangian

$$\begin{aligned} \mathcal{L}_{GNLSM} = & -\frac{1}{2} \operatorname{tr} (W_{\mu\nu} W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{4} v^2 \operatorname{tr} [(D_\mu U)^\dagger (D^\mu U)] \\ & + \text{fermionic terms}, \end{aligned} \quad (2.8)$$

which is also gauge invariant but nonrenormalizable. Like in the linear case, nonstandard interactions can be parametrized by adding gauge invariant terms of higher dimension⁵, in

⁵Usually, within nonlinear theories the dimension is counted differently [6, 13, 14, 15], since U (2.7) has dimension 0, while the linear expression Φ (2.1d) has dimension 1. However, for consistency, we count the dimension of nonlinear terms like in the linear case, thereby attributing the dimension 1 to U , which is justified, since U occurs together with the vacuum expectation value v in (2.6). We will see that our counting method makes sense when classifying the terms concerning the high-energy behaviour they yield.

which the scalar sector is now nonlinearly realized as (2.7), to the GNLSM Lagrangian. The resulting Lagrangian

$$\mathcal{L}_{eff}^{NL} = \mathcal{L}_{GNLSM} + \sum_{n>4} \sum_i \frac{\epsilon_i}{\Lambda^{n-4}} \mathcal{L}_i^{(n)} \quad (2.9)$$

is called a chiral Lagrangian. Also the loop effects within the GNLSM generate such effective interaction terms [6].

In this article we will consider the nonlinear effective terms corresponding to (2.4a) – (2.4c) and (2.5a) – (2.5d). They are obtained from the linear terms by the substitution (2.6). Only (2.4d) – (2.4f) can be neglected in this case because by a field and coupling constant redefinition they can be completely absorbed into the GNLSM Lagrangian. We name the nonlinear terms like the linear ones, since we will treat linear and nonlinear Lagrangians separately and no confusion can occur.

One could assume that a third possibility to construct effective Lagrangians exists, namely to construct Lagrangians that are not gauge invariant (except for the electromagnetic gauge freedom). This possibility, however, is artificial because each gauge noninvariant Lagrangian can be written as a (nonlinear) gauge theory such as (2.9) by means of a Stueckelberg transformation [19, 20, 21]. This means that after applying a Stueckelberg transformation in order to introduce unphysical scalar partners of the massive vector fields, one can even apply the GET in order to simplify calculations within originally gauge noninvariant theories.

One can easily convince oneself that the GET (1.2) is also valid for effective gauge theories of the type (2.2) or (2.9). In [2] only the invariance of the quantized Lagrangian under BRS transformations is used in order to prove the GET. Renormalizability is not required in the derivation. Since the additional effective interaction terms are also gauge invariant and thus BRS invariant, they do not affect the validity of the proof. However the explicit form of the BRS transformations is used in [2]. This is not changed by adding effective interaction terms to the Lagrangian but by parametrizing the scalar sector nonlinearly as (2.6). Thus, the proof in [2] only holds for linear effective Lagrangians. An analogous proof for the nonlinear case can be found in [7]. The general validity of the GET can most easily be seen from [3] where BRS invariance itself is used but not the explicit form of the BRS transformations; thus the proof in [3] applies to linear and nonlinear effective gauge theories.

In order to apply power counting in the next section, we now determine for each vertex deriving from the each of the effective interaction terms the power of the energy on which its vertex factor depends. Obviously, this power is identical to the number of derivatives in the corresponding expression within the effective term⁶, which is easily found by substituting the definitions (2.1a) – (2.1e) and (2.7) into the in the effective term and then taking the traces. We only consider vertices that contribute — after applying the GET (1.2) — to the amplitudes for $VV \rightarrow VV$ and $f\bar{f} \rightarrow VV$ (in the tree approximation) .

Table 1 lists the (types of) vertices that are of interest for us and shows which vertices derive from each effective interaction term and on which power of E the corresponding vertex factor depends. The first part contains the SM vertices and the additional contribution from the linear effective interaction term; the second part contains the vertices from the GNLSM Lagrangian and from the corresponding nonlinear effective terms. For better comparison, lines and columns corresponding to interaction terms and to vertices that do not occur in

⁶Even within effective theories the Feynman rules can directly be obtained from the effective Lagrangian in the usual way [22]. This relies on the equivalence of Hamiltonian and Lagrangian path integral quantization.

the nonlinear case are left free. Terms that have a similar structure are put together in one column, e.g. $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$ are in one column denoted as $\mathcal{L}_{W\Phi, B\Phi}$, etc. It should be noted that in general not all vertices of a given type exist. For instance, VVV vertices with three neutral vector bosons do not exist, $\mathcal{L}_{B\Phi}$ implies only vertices with at least one neutral vector boson, \mathcal{L}_D , \mathcal{L}'_{DD1} and \mathcal{L}'_{DD2} yield no vertices with photons, etc.

The effective terms yield higher powers of E in the vertices or even induce vertices not present in the SM. We will see in the next section that, as one expects, this will imply a worse high-energy behaviour of S -matrix elements.

3 The Power-Counting Method

In this section we will apply the GET (1.2) in order to analyze the high-energy behaviour of scattering processes in the presence of the effective interactions introduced above. This will yield many explicit examples for the invalidity of the ET within effective theories. We will show that the GET simplifies calculations of S -matrix elements because no cancellations occur.

In order to determine the high-energy behaviour (i.e. the highest power of E occurring in the amplitude) of a process with external longitudinally polarized vector bosons within an effective theory and to calculate the leading term of the corresponding S -matrix element (in the tree approximation) we proceed as follows: We apply the GET (1.2) in order to express such an S -matrix element as the sum of all amplitudes (calculated within the R_ξ -gauge) in which each longitudinal vector boson is either replaced by the corresponding Goldstone boson or its polarization vector by the nonleading term v (1.3) (multiplied by a phase factor). Then we construct all Feynman diagrams that contribute to these amplitudes and determine the high-energy behaviour of each diagram. We neglect all effects that are proportional to $\epsilon_i \epsilon_j$ with ϵ_i and ϵ_j being effective coupling constants associated with the higher dimension terms (see (2.2) and (2.9)), i.e. in all exchange diagrams only one vertex is taken from an effective interaction term but the other one from the SM or the GNLSM, respectively. The high-energy behaviour of the various diagrams can be determined by simple power counting using Table 1. (An external v line is $O(E^{-1})$ due to the definition (1.3).) When applying power counting within the linear models we always assume that $E \gg M_H$; the heavy-Higgs scenario is parametrized by the nonlinear models.

Tables 2a – 2e and 3a – 3c display the results of this power-counting method applied to $VV \rightarrow VV$ and $f\bar{f} \rightarrow VV$ for all effective interaction terms introduced in Section 2. We have considered all possible combinations of polarizations of the vector bosons. (Processes with all V s being transversal have only been considered for completeness.) The following shorthand notation for the (types of) Feynman diagrams was used: The symbols outside the brackets denote the external lines and those inside the brackets the exchanged particles such that symbols written on the same side of the bracket denote particles coupled the same end of the propagator⁷. (C) stands for a four-particle contact interaction. The tables display in the first part the high-energy behaviour of the various Feynman diagrams within the SM and the additional contributions of the linear effective interaction terms. The second part shows

⁷ It should be noted, that e.g. $vv(V)\phi\phi$ corresponds to $vv \rightarrow \phi\phi$ or $\phi\phi \rightarrow vv$ if the vector boson is exchanged in the s -channel but to $v\phi \rightarrow v\phi$ if the vector boson is exchanged in the t - or u -channel; due to the phase factor in (1.2) both contributions have a relative factor -1 .

the high-energy behaviour within the GNLSM and the contributions of the corresponding nonlinear effective terms.

From these tables one can directly read off the high-energy behaviour induced by a given interaction term and which diagrams contain the leading contributions. Then one only has to calculate these diagrams in order to obtain the correct S -matrix element up to nonleading terms. Thus, having once found the vertices implied by the effective terms (2.4a) – (2.4f) and (2.5a) – (2.5c), our method enables one to determine the high-energy behaviour of a process within a given effective theory simply by power counting without doing any calculation, and it simplifies the calculation of the corresponding amplitudes since only the leading parts of the single diagrams have to be considered. As mentioned in the introduction, power counting cannot be applied if the GET (1.2) is not used and diagrams with external V_L s have to be calculated, since cancellations occur.

However, from the tables it is obvious that the ET (1.1) is not valid for effective theories, because, in general, the leading contributions of a given term do not only (or not at all) stem from diagrams in which all V_L s are replaced by ϕ s but also from diagrams with external vs . In order to calculate these, one should keep in mind that the leading part of v^μ (1.3) has the simple form

$$v_\mu = -\frac{M}{2P_0^2}(P_0, -P_i) + O(E^{-3}). \quad (3.1)$$

If the in- or outgoing state consist of two particles (1 and 2) with the same mass one finds in the CM system [23]

$$v_1^\mu = -\frac{2M}{E^2}P_2^\mu + O(E^{-3}). \quad (3.2)$$

(with E being the total energy) and vice versa for v_2^μ .

In order to give an example how our method works, let us look more closely at the process $V_L V_L \rightarrow V_L V_L$ (Table 2a). By applying power counting to the diagrams that have to be considered in a direct calculation without using the GET (1.2), namely $V_L V_L V_L V_L(C)$, $V_L V_L(H, \phi) V_L V_L$ and $V_L V_L(V) V_L V_L$, one would expect that the amplitudes induced by the various terms diverge as $O(E^4)$, the contributions of \mathcal{L}_W and \mathcal{L}'_W even as $O(E^6)$. However, all amplitudes except for those stemming from \mathcal{L}'_{DD1} and \mathcal{L}'_{DD2} behave at most as $O(E^2)$, the effects of \mathcal{L}_W , \mathcal{L}_{WW} , \mathcal{L}_{BB} and \mathcal{L}'_W are even $O(E^0)$. Within a direct calculation this high-energy behaviour is the result of cancellations; however it is obvious from the GET (1.2). Furthermore, Table 2a shows that for most of the effective terms the ET is not valid because the leading part of the amplitude gets contributions from diagrams with vs , e.g. for \mathcal{L}_W and \mathcal{L}'_W it is given by the $vv(V)\phi\phi$ diagrams while the diagrams with four external ϕ s yield no contributions. (For $V_T V_L \rightarrow V_L V_L$ (Table 2b), the ET even would imply that these two terms do not affect the good high-energy behaviour of the SM; in fact they yield $O(E^1)$ divergences.)

It may happen in some very special cases that the high-energy behaviour is even better than expected from the tables, because the Feynman diagrams that would yield the leading contribution do not exist or cancel accidentally. We illustrate this by considering the effects of \mathcal{L}_W , $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$ on processes of the type $V_T V_T \rightarrow V_L V_L$. Table 2c predicts that the contributions of these terms behave as $O(E^2)$ which is indeed the case for most of these processes [12]. Let us however look closer at $W_T^+ W_T^+ \rightarrow W_L^+ W_L^+$. The $O(E^2)$ contribution of \mathcal{L}_W would come from $W_T^+ W_T^+(V)\phi^+\phi^+$ diagrams, which do not exist in this case. For $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$ also the $W_T^+ W_T^+\phi^+\phi^+(C)$ and $W_T^+\phi^+(H, \phi_0)W_T^+\phi^+$ diagrams should contribute. However, it turns out that $\mathcal{L}_{B\Phi}$ affects none of these diagrams (it yields only vertices with

at least one neutral vector boson), $\mathcal{L}_{W\Phi}$ only contributes to $W_T^+\phi^+(H, \phi_0)W_T^+\phi^+$ and the $O(E^2)$ part of this contribution cancels. Thus all three terms yield at most $O(E^0)$ corrections to $W_T^+W_T^+ \rightarrow W_L^+W_L^+$ [12]. Another accidental cancellation is indicated in the footnote of Table 2b.

The leading contributions of \mathcal{L}_W , $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$ (as long as they are at least $O(E^1)$) on $VV \rightarrow VV$ were calculated directly without applying the GET in [12]. We have compared our results with those of [12] and found that they agree.

The phenomenologically most important results of our analysis are the following:

- All dimension-six terms (2.4a) – (2.4f) yield at most $O(E^2)$ contributions to the S -matrix elements and thus, in the linear case, only $O(E^0)$ contributions to the total cross-sections⁸ ($\sigma \propto \mathcal{M}^2/E^2$). I.e., they only slightly affect the good high-energy behaviour of the SM. This result has been obtained for \mathcal{L}_W , $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$ in [12], we find it for all dimension-six terms.
- The dimension-eight terms (2.5a) – (2.5c) are supposed to yield a worse high energy behaviour because of their higher dimension. This is indeed the case for \mathcal{L}'_{DD1} and \mathcal{L}'_{DD2} that imply $O(E^4)$ corrections to the S -matrix elements. The terms \mathcal{L}'_W and $\mathcal{L}'_{W\Phi}$ however only yield $O(E^2)$ effects on the amplitudes⁹, i.e. they behave in the same manner as the dimension-six terms.
- For most of the effective terms the worst high-energy behaviour is associated with external longitudinal vector bosons, as one expects. However the quadrupole terms \mathcal{L}_W and \mathcal{L}'_W behave well in $V_LV_L \rightarrow V_LV_L$ and $f\bar{f} \rightarrow V_LV_L$ but yield increasing amplitudes for $V_TV_T \rightarrow V_TV_T$ and $f\bar{f} \rightarrow V_TV_T$ [10, 11, 12]. Furthermore, the contribution of $\mathcal{L}_{W\Phi}$, $\mathcal{L}_{B\Phi}$ and $\mathcal{L}'_{W\Phi}$ to $V_TV_TV_LV_L$ -amplitudes is as divergent as the one to $V_LV_LV_LV_L$ -amplitudes, namely $O(E^2)$ [12]. Thus it is not sufficient to study only longitudinally polarized initial and final states when looking for effects of new physics in vector-boson scattering.
- While in the SM the presence of a (light) Higgs boson improves the high-energy behaviour (in the (Higgs-less) GNLSM $V_LV_LV_LV_L$ -amplitudes are $O(E^2)$ but in the SM they are $O(E^0)$ for $E \gg M_H$), this is not the case for the higher-dimension extensions. The high-energy behaviour of the effective terms is the same in the linear case (with a light Higgs boson) and in the nonlinear case (without a Higgs boson). Actually, the value of the leading part of an S -matrix element is affected by the presence of the Higgs boson but the largest occurring power of E is unchanged. For \mathcal{L}_W , $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$ this has already been found in [12].

Our power-counting method is even useful within the SM if one considers processes for which the amplitude is only $O(E^{-1})$ (see Tables 2b, 2d and 3b). Although the ET (1.1) is formally correct in this case, it is obviously of no use, because the corrections neglected in (1.1) are of the same order as the amplitude itself. Applying the GET (1.2) one can correctly determine the leading (i.e. $O(E^{-1})$) part of the amplitude. Again this is not given by diagrams with all V_L s being replaced by ϕ s alone.

⁸ Remember that we neglect $\epsilon_i\epsilon_j$ -terms.

⁹There is one exception, namely \mathcal{L}'_W yields an $O(E^3)$ effect on $W_L^\pm W_T^\mp Z_T \gamma$ amplitudes (see Table 2d). However, the amplitude in the SM or in the GNLSM is $O(E^{-1})$ and thus the contribution to the cross-section is only $O(E^0)$.

Although the ET is in general not valid within effective theories, there is an important exception. Since the ET correctly determines the leading amplitudes within the SM for $E \ll M_H$ [4], it is also valid for the leading parts of S -matrix elements calculated within the GNLSM¹⁰ (without additional terms of higher dimension), which is the heavy-Higgs limit of the SM. Furthermore, the leading tree-level contribution to the special process $V_L V_L \rightarrow V_L V_L$ stems from the $\phi\phi\phi\phi(C)$ diagram (see Table 2a), i.e. it is given by a pure scalar self interaction. In [24] it has been shown that even the leading one-loop corrections to this process stem from diagrams in which all internal lines are scalar lines. Therefore, *in this special case* even no internal vector lines have to be considered and the leading amplitude can be obtained from a Lagrangian which contains only scalar fields, namely the one that is found by dropping all vector fields in the GNLSM Lagrangian (2.8):

$$\mathcal{L}_{\text{scalar}} = \frac{1}{4}v^2 \text{tr} [(\partial_\mu U)^\dagger (\partial^\mu U)] \quad (3.3)$$

(nonlinear σ model). Since the GNLSM is the heavy-Higgs limit of the SM, the analogous statement is true within the SM for $E \ll M_H$; the leading tree-level contribution to $V_L V_L \rightarrow V_L V_L$ comes from the $\phi\phi\phi\phi(C)$ and $\phi\phi(H)\phi\phi$ diagrams in this case¹¹ [2, 4].

4 Comparison with the Literature

After having shown how the equivalence theorem can be generalized such that it even applies to effective Lagrangians we now critically analyze those articles where the ET in the form (1.1), which is in general not valid beyond the SM, was used within effective theories. We compare this treatment with the results of the power-counting method described in the previous section and examine whether the a priori incorrect use of (1.1) led to wrong results or not.

In [20, 24, 25] the ET was used within the GNLSM in order to determine the amplitudes for $V_L V_L \rightarrow V_L V_L$ in the tree approximation and at the one-loop level. Furthermore diagrams with internal vector lines were neglected. As discussed in the previous section this procedure is correct because no additional nonstandard interaction terms were considered. In [26] the same processes was investigated within the BESS model (which is an extension of the GNLSM with one additional heavy gauge-boson triplet) by applying the ET; this is also correct.

In [13, 14, 15] the ET was applied to Lagrangians with additional effective interaction terms of higher dimension. In all these articles nonlinear effective Lagrangians of the type (2.9), i.e. Lagrangians without a Higgs boson were considered. In [14] the contributions of $\mathcal{L}_{W\Phi}$ (2.4b) and $\mathcal{L}_{B\Phi}$ (2.4c) to the amplitude for $q\bar{q} \rightarrow VV$ were calculated by using the ET. As one can see from Tables 3a – 3c, this is correct; the leading contributions to the amplitudes are indeed found by replacing all longitudinal vector bosons by Goldstone bosons. In [15], the amplitudes for $q\bar{q} \rightarrow V_L V_L$, $V_L V_L \rightarrow V_L V_L$, $V_T V_L \rightarrow V_L V_L$ and $V_T V_T \rightarrow V_L V_L$, were calculated within the GNLSM at the one-loop level and in addition the tree-level contributions of the (nonlinear versions of) the effective terms $\mathcal{L}_{W\Phi}$ (2.4b), $\mathcal{L}_{B\Phi}$ (2.4c),

¹⁰Note, however, that the relation (1.1) is *not* valid. The leading (i.e. $O(E^2)$) tree-level contribution to $V_L V_L \rightarrow V_L V_L$ stems from the $\phi\phi\phi\phi(C)$ diagram and can thus be determined by applying (1.1). However the corrections from diagrams with vs are $O(E^0)$ (see Table 2a) and not only $O(E^{-1})$ as in (1.1).

¹¹However, note that for $E \gg M_H$ the vector exchange diagrams $\phi\phi(V)\phi\phi$ must not be neglected.

\mathcal{L}'_{DD1} (2.5c) and \mathcal{L}'_{DD2} (2.5d) (and of two further effective terms that we did not consider because they affect four-fermion amplitudes at the tree level) were determined by applying the ET. This procedure is wrong because the ET is not valid for the terms $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$. Furthermore, it seems as if only contact diagrams were calculated (see Figure 2 there). To discuss this in more detail, let us consider the effects of $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$ on $Z_L Z_L \rightarrow W_L^+ W_L^-$. In [15], no $O(E^2)$ -contributions of these terms were found (see Appendix there), however such contributions exist, as one can see from Table 2a. (Also see [12].) They stem from the $\phi\phi(V)\phi\phi$ - and the $v\phi\phi\phi(C)$ diagrams. If one applies the ET (1.1), one neglects the $v\phi\phi\phi(C)$ contribution and obtains a wrong result. If one even considers only contact diagrams, one finds no $O(E^2)$ effects of these terms at all, since they do not affect the $\phi\phi\phi\phi(C)$ diagrams.

In [13] the one-loop effects of the GNLSM and the tree-level effects of effective higher-dimension terms on $V_L V_L \rightarrow V_L V_L$ were calculated, too. In those articles not only the ET was used but also internal vector boson lines were neglected, i.e. the GNLSM was parametrized by the pure scalar Lagrangian (3.3) and also in the effective interaction terms all vector fields were set equal to zero. As discussed in Section 3 this procedure is correct for the GNLSM. However the treatment some of the additional effective interaction terms in [13] is wrong. Terms like $\mathcal{L}_{W\Phi}$ (2.4b) and $\mathcal{L}_{B\Phi}$ (2.4c) were not considered at all because they vanish for vanishing gauge fields; their effects are simply forgotten if one tries to describe effective interactions of vector bosons by pure scalar self-interactions.

Thus we have found that in the articles, in which the ET (1.1) was applied to effective Lagrangians, the leading parts of S -matrix elements calculated within the GNLSM were correctly reproduced (for $V_L V_L \rightarrow V_L V_L$ even if internal vector lines were neglected), but incorrect results were obtained for the contributions of several additional effective interaction terms of higher dimension.

It should be noted, however, that the numerical effects of this incorrect use of the ET are rather small as long as the coefficients of all effective interaction terms are assumed to be of the same order of magnitude. To explain this let us again consider $V_L V_L \rightarrow V_L V_L$ (Table 2a). If one applies the ET, the contributions of $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$ are not correctly reproduced but the correct contributions of \mathcal{L}'_{DD1} and \mathcal{L}'_{DD2} are found. The contributions of the first two terms are $O(E^2)$, while those of the latter ones are $O(E^4)$. Thus, if the coefficients of all terms are of the same order of magnitude, the correctly determined effects of \mathcal{L}'_{DD1} and \mathcal{L}'_{DD2} are larger than the incorrectly determined effects of $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$. In $V_T V_T \rightarrow V_L V_L$ (Table 2c) all four terms yield $O(E^2)$ effects, but in this case the ET is also valid for $\mathcal{L}_{W\Phi}$ and $\mathcal{L}_{B\Phi}$. However, if one assumes that the coefficients of the dimension-eight terms are much smaller than those of the dimension-six terms, because they are suppressed by higher negative powers of the scale of the new physics Λ [5, 12, 16, 18] (see (2.2) and (2.9)) the errors in these works become relevant and their results cannot be applied.

5 Summary

In this article we have pointed out that the equivalence theorem cannot be applied to effective Lagrangians, which contain nonstandard interaction terms of higher dimension in addition to the Lagrangian of the standard model or of the gauged nonlinear σ model.

We have developed a correct method to simplify calculations of S -matrix elements for scattering processes with external longitudinally polarized vector bosons within effective theories: Instead of applying the equivalence theorem (1.1), one has to use the generalized

equivalence theorem (1.2), which also holds for effective Lagrangians. On the basis of this theorem one can determine the high-energy behaviour of S -matrix elements simply by power counting without doing any calculations and the corresponding amplitudes can be calculated more easily since no cancellations occur.

We have applied this power-counting method to the phenomenologically most important dimension-six and dimension-eight effective interaction terms in order to analyze their high-energy behaviour in $VV \rightarrow VV$ and in $f\bar{f} \rightarrow VV$. We have found that all dimension-six terms and even some of the dimension-eight terms only slightly affect the good high-energy behaviour of the standard model, because their contributions to the S -matrix elements are at most $O(E^2)$ and not $O(E^4)$ as one would expect. Furthermore we have shown that the addition of a light Higgs boson does not improve the high-energy behaviour of these effective terms.

Concerning the importance of our power-counting method we adopt a statement made in [23] for the equivalence theorem itself, namely that its true value is not the simplification of calculations, because everything can also be calculated directly without applying the generalized equivalence theorem (1.2) (as in [12]). Its true value is that it gives a deep insight in the qualitative nature of the effects of nonstandard interaction terms on S -matrix elements, which is not directly obvious because cancellations occur in a straightforward calculation. Of course, if one calculates all S -matrix elements, the high-energy behaviour of the contributions of the various effective terms can also be determined and the conclusions of the previous paragraph can also be drawn, however we have obtained these qualitative results without calculations.

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Tables

Vertex	Linear Lagrangians							
	\mathcal{L}_{SM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi,B\Phi}$	$\mathcal{L}_{WW,BB}^a$	\mathcal{L}_D^a	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$	$\mathcal{L}_{DD1,2}$
$\phi\phi\phi\phi$	0	—	—	—	2	—	—	4
$V\phi\phi\phi$	—	—	—	—	1	—	3	3
$VV\phi\phi$	0	—	2	2	0	—	2	2
$VVV\phi$	—	—	1	—	—	3^b	1	1
$VVVV$	0	2	0^c	—	—	2	0	0
$\phi\phi\phi$	—	—	—	—	—	—	—	—
$H\phi\phi$	0	—	—	—	2	—	—	—
$V\phi\phi$	1	—	3	—	—	—	3	—
$VH\phi$	1	—	3	—	1	—	3	—
$VV\phi$	0	—	2	—	—	—	2	—
VVH	0	—	2	2	0	—	2	—
VVV	1	3	1	—	—	3	1	—
$f\bar{f}V$	0	—	—	—	—	—	—	—

Vertex	Nonlinear Lagrangians							
	\mathcal{L}_{GNLSM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi,B\Phi}$	—	—	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$	$\mathcal{L}_{DD1,2}$
$\phi\phi\phi\phi$	2	—	—			—	—	4
$V\phi\phi\phi$	1	—	3			—	3	3
$VV\phi\phi$	0	—	2			—	2	2
$VVV\phi$	—	—	1			3^b	1	1
$VVVV$	0	2	0^c			2	0	0
$\phi\phi\phi$	—	—	—			—	—	—
—								
$V\phi\phi$	1	—	3			—	3	—
—								
$VV\phi$	0	—	2			—	2	—
—								
VVV	1	3	1			3	1	—
$f\bar{f}V$	0	—	—			—	—	—

^aAfter a field and coupling constant redefinition

^bOnly $\gamma ZW^\pm\phi^\mp$ vertices

^cOnly $\mathcal{L}_{W\Phi}$

Table 1: Vertices deriving from the effective interaction terms, which contribute to $VV \rightarrow VV$ and to $f\bar{f} \rightarrow VV$ after applying the GET (1.2). The integers denote the powers of the energy E in the vertex factors.

$V_L V_L V_L V_L$								
Diagram	Linear Lagrangians							
	\mathcal{L}_{SM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi,B\Phi}$	$\mathcal{L}_{WW,BB}$	\mathcal{L}_D	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$	$\mathcal{L}'_{DD1,2}$
$\phi\phi\phi\phi(C)$	$\boxed{0}$	—	—	—	$\boxed{2}$	—	—	$\boxed{4}$
$\phi\phi(H)\phi\phi$	—2	—	—	—	0	—	—	—
$\phi\phi(V)\phi\phi$	$\boxed{0}$	—	$\boxed{2}$	—	—	—	$\boxed{2}$	—
$v\phi\phi\phi(C)$	—	—	—	—	0	—	$\boxed{2}$	2
$v\phi(H)\phi\phi$	—2	—	0	—	0	—	0	—
$v\phi(V)\phi\phi$	—2	—	0	—	—	—	0	—
$vv\phi\phi(C)$	—2	—	0	$\boxed{0}$	—2	—	0	0
$vv(H)\phi\phi$	—4	—	—2	—2	—2	—	—2	—
$v\phi(H,\phi)v\phi$	—2	—	0	—	—2	—	0	—
$vv(V)\phi\phi$	—2	$\boxed{0}$	0	—	—	$\boxed{0}$	0	—
$v\phi(V)v\phi$	—4	—	—2	—	—	—	—2	—
$vvv\phi(C)$	—	—	—2	—	—	a	—2	—2
$vv(H,\phi)v\phi$	—4	—	—2	—2	—4	—	—2	—
$vv(V)v\phi$	—4	—2	—2	—	—	—2	—2	—
$vvvv(C)$	—4	—2	—4	—	—	—2	—4	—4
$vv(H,\phi)vv$	—6	—	—4	—4	—6	—	—4	—
$vv(V)vv$	—4	—2	—4	—	—	—2	—4	—
	×		×		×			×
Diagram	Nonlinear Lagrangians							
	\mathcal{L}_{GNLSM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi,B\Phi}$	—	—	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$	$\mathcal{L}'_{DD1,2}$
$\phi\phi\phi\phi(C)$	$\boxed{2}$	—	—			—	—	$\boxed{4}$
—								
$\phi\phi(V)\phi\phi$	0	—	$\boxed{2}$			—	$\boxed{2}$	—
$v\phi\phi\phi(C)$	0	—	$\boxed{2}$			—	$\boxed{2}$	2
—								
$v\phi(V)\phi\phi$	—2	—	0			—	0	—
$vv\phi\phi(C)$	—2	—	0			—	0	0
—								
$v\phi(\phi)v\phi$	—2	—	0			—	0	—
$vv(V)\phi\phi$	—2	$\boxed{0}$	0			$\boxed{0}$	0	—
$v\phi(V)v\phi$	—4	—	—2			—	—2	—
$vvv\phi(C)$	—	—	—2			a	—2	—2
$vv(\phi)v\phi$	—4	—	—2			—	—2	—
$vv(V)v\phi$	—4	—2	—2			—2	—2	—
$vvvv(C)$	—4	—2	—4			—2	—4	—4
$vv(\phi)vv$	—6	—	—4			—	—4	—
$vv(V)vv$	—4	—2	—4			—2	—4	—
	×							×

^aThe $VVV\phi$ vertex stemming from \mathcal{L}'_W yields no contribution in this case, because one V is a photon, which cannot be longitudinal.

Table 2a: The leading powers of the Energy E in the contribution (linear in the ϵ_i) of the effective interaction terms to the Feynman diagrams that yield $\mathcal{M}(V_L V_L \rightarrow V_L V_L)$ after applying the GET (1.2). The leading contribution of each term is framed. The crosses denote the terms for which the leading diagrams are only those with all external V_L s being replaced by ϕ s (written above the horizontal line). The shorthand notation for the (types of) Feynman diagrams is explained in the text.

$V_T V_L V_L V_L$								
Diagram	Linear Lagrangians							
	\mathcal{L}_{SM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi,B\Phi}$	$\mathcal{L}_{WW,BB}$	\mathcal{L}_D	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$	$\mathcal{L}'_{DD1,2}$
$V_T \phi \phi \phi(C)$	–	–	–	–	$\boxed{1}$	–	$\boxed{1^a}$	$\boxed{3}$
$V_T \phi(H) \phi \phi$	$\boxed{-1}$	–	$\boxed{1}$	–	$\boxed{1}$	–	$\boxed{1}$	–
$V_T \phi(V) \phi \phi$	$\boxed{-1}$	–	$\boxed{1}$	–	–	–	$\boxed{1}$	–
$V_T v \phi \phi(C)$	$\boxed{-1}$	–	$\boxed{1}$	$\boxed{1}$	–1	–	$\boxed{1}$	1
$V_T v(H) \phi \phi$	–3	–	–1	–1	–1	–	–1	–
$V_T \phi(H, \phi) v \phi$	$\boxed{-1}$	–	$\boxed{1}$	–	–1	–	$\boxed{1}$	–
$V_T v(V) \phi \phi$	$\boxed{-1}$	$\boxed{1}$	$\boxed{1}$	–	–	$\boxed{1}$	$\boxed{1}$	–
$V_T \phi(V) v \phi$	–3	–	–1	–	–	–	–1	–
$V_T v v \phi(C)$	–	–	–1	–	–	$\boxed{1}$	–1	–1
$V_T v(H, \phi) v \phi, vv(H, \phi) V_T \phi$	–3	–	–1	–1	–3	–	–1	–
$V_T v(V) v \phi, vv(V) V_T \phi$	–3	–1	–1	–	–	–1	–1	–
$V_T v v v(C)$	–3	–1	–3	–	–	–1	–3	–3
$V_T v(H, \phi) v v$	–5	–	–3	–3	–5	–	–3	–
$V_T v(V) v v$	–3	–1	–3	–	–	–1	–3	–
					×			×
Diagram	Nonlinear Lagrangians							
	\mathcal{L}_{GNLSM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi,B\Phi}$	–	–	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$	$\mathcal{L}'_{DD1,2}$
$V_T \phi \phi \phi(C)$	$\boxed{1}$	–	$\boxed{1^a}$			–	$\boxed{1^a}$	$\boxed{3}$
–								
$V_T \phi(V) \phi \phi$	–1	–	$\boxed{1}$			–	$\boxed{1}$	–
$V_T v \phi \phi(C)$	–1	–	$\boxed{1}$			–	$\boxed{1}$	1
–								
$V_T \phi(\phi) v \phi$	–1	–	$\boxed{1}$			–	$\boxed{1}$	–
$V_T v(V) \phi \phi$	–1	$\boxed{1}$	$\boxed{1}$			$\boxed{1}$	$\boxed{1}$	–
$V_T \phi(V) v \phi$	–3	–	–1			–	–1	–
$V_T v v \phi(C)$	–	–	–1			$\boxed{1}$	–1	–1
$V_T v(\phi) v \phi, vv(\phi) V_T \phi$	–3	–	–1			–	–1	–
$V_T v(V) v \phi, vv(V) V_T \phi$	–3	–1	–1			–1	–1	–
$V_T v v v(C)$	–3	–1	–3			–1	–3	–3
$V_T v(\phi) v v$	–5	–	–3			–	–3	–
$V_T v(V) v v$	–3	–1	–3			–1	–3	–
	×							×

^aBy power counting these diagrams are $O(E^3)$, but all $O(E^3)$ terms cancel.

Table 2b: Same as Table 2a for vector-boson scattering with one transversal and three longitudinal external vector bosons: $V_T V_L \rightarrow V_L V_L$ and $V_L V_L \rightarrow V_T V_L$.

$V_T V_T V_L V_L$								
Diagram	Linear Lagrangians							
	\mathcal{L}_{SM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi,B\Phi}$	$\mathcal{L}_{WW,BB}$	\mathcal{L}_D	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$	$\mathcal{L}'_{DD1,2}$
$V_T V_T \phi \phi(C)$	$\boxed{0}$	—	$\boxed{2}$	$\boxed{2}$	$\boxed{0}$	—	$\boxed{2}$	$\boxed{2}$
$V_T V_T(H) \phi \phi$	—2	—	0	0	$\boxed{0}$	—	0	—
$V_T \phi(H, \phi) V_T \phi$	$\boxed{0}$	—	$\boxed{2}$	—	—	—	$\boxed{2}$	—
$V_T V_T(V) \phi \phi$	$\boxed{0}$	$\boxed{2}$	$\boxed{2}$	—	—	$\boxed{2}$	$\boxed{2}$	—
$V_T \phi(V) V_T \phi$	—2	—	0	—	—	—	0	—
$V_T V_T v \phi(C)$	—	—	0	—	—	$\boxed{2}$	0	0
$V_T V_T(H, \phi) v \phi, V_T v(H, \phi) V_T \phi$	—2	—	0	0	—2	—	0	—
$V_T V_T(V) v \phi, V_T v(V) V_T \phi$	—2	0	0	—	—	0	0	—
$V_T V_T v v(C)$	—2	0	—2	—	—	0	—2	—2
$V_T V_T(H, \phi) v v, V_T v(H, \phi) V_T v$	—4	—	—2	—2	—4	—	—2	—
$V_T V_T(V) v v, V_T v(V) V_T v$	—2	0	—2	—	—	0	—2	—
	\times	\times	\times	\times	\times	\times	\times	\times

Table 2c: Same as Table 2a for vector-boson scattering with two transversal and two longitudinal external vector bosons: $V_T V_T \rightarrow V_L V_L$, $V_T V_L \rightarrow V_T V_L$ and $V_L V_L \rightarrow V_T V_T$. Only the linear effective terms are listed because in the nonlinear case the corresponding powers of E are the same.

$V_T V_T V_T V_L$								
Diagram	\mathcal{L}_{SM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi,B\Phi}$	$\mathcal{L}_{WW,BB}$	\mathcal{L}_D	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$	$\mathcal{L}'_{DD1,2}$
$V_T V_T V_T \phi(C)$	—	—	$\boxed{1}$	—	—	$\boxed{3^a}$	$\boxed{1}$	$\boxed{1}$
$V_T V_T(H, \phi) V_T \phi$	$\boxed{-1}$	—	$\boxed{1}$	$\boxed{1}$	$\boxed{-1}$	—	$\boxed{1}$	—
$V_T V_T(V) V_T \phi$	$\boxed{-1}$	$\boxed{1}$	$\boxed{1}$	—	—	1	$\boxed{1}$	—
$V_T V_T V_T v(C)$	$\boxed{-1}$	$\boxed{1}$	—1	—	—	1	—1	—1
$V_T V_T(H, \phi) V_T v$	—3	—	—1	—1	—3	—	—1	—
$V_T V_T(V) V_T v$	$\boxed{-1}$	$\boxed{1}$	—1	—	—	1	—1	—
			\times	\times	\times	\times	\times	\times

^aOnly $W_L^\pm W_T^\mp Z_T \gamma$ reactions

Table 2d: Same as Table 2c for vector-boson scattering with three transversal and one longitudinal external vector boson: $V_T V_T \rightarrow V_T V_L$ and $V_T V_L \rightarrow V_T V_T$.

$V_T V_T V_T V_L$								
Diagram	\mathcal{L}_{SM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi,B\Phi}$	$\mathcal{L}_{WW,BB}$	\mathcal{L}_D	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$	$\mathcal{L}'_{DD1,2}$
$V_T V_T V_T V_T(C)$	$\boxed{0}$	$\boxed{2}$	$\boxed{0}$	—	—	$\boxed{2}$	$\boxed{0}$	$\boxed{0}$
$V_T V_T(H, \phi) V_T V_T$	—2	—	$\boxed{0}$	$\boxed{0}$	$\boxed{-2}$	—	$\boxed{0}$	—
$V_T V_T(V) V_T V_T$	$\boxed{0}$	$\boxed{2}$	$\boxed{0}$	—	—	$\boxed{2}$	$\boxed{0}$	—

Table 2e: Same as Table 2c for $V_T V_T \rightarrow V_T V_T$.

$f\bar{f} \rightarrow V_L V_L$					
Diagram	\mathcal{L}_{SM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi, B\Phi}$	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$
$f\bar{f}(V)\phi\phi$	$\boxed{0}$	—	$\boxed{2}$	—	$\boxed{2}$
$ff(V)v\phi$	—2	—	0	—	0
$f\bar{f}(V)vv$	—2	$\boxed{0}$	—2	$\boxed{0}$	—2
$fv(f)\bar{f}v$	—2	—	—	—	—
	\times		\times		\times

Table 3a: Same as Table 2c for $f\bar{f} \rightarrow V_L V_L$. Diagrams with $f\bar{f}H$ and $f\bar{f}\phi$ vertices are not considered because they are negligible for light fermions. The interaction terms that are not listed yield no contribution.

$f\bar{f} \rightarrow V_T V_L$					
Diagram	\mathcal{L}_{SM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi, B\Phi}$	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$
$ff(V)V_T\phi$	$\boxed{-1}$	—	$\boxed{1}$	—	$\boxed{1}$
$f\bar{f}(V)V_T v$	$\boxed{-1}$	$\boxed{1}$	—1	$\boxed{1}$	—1
$fV_T(f)\bar{f}v, fv(f)\bar{f}V_T$	$\boxed{-1}$	—	—	—	—
			\times		\times

Table 3b: Same as Table 3a for $f\bar{f} \rightarrow V_T V_L$.

$f\bar{f} \rightarrow V_T V_T$					
Diagram	\mathcal{L}_{SM}	\mathcal{L}_W	$\mathcal{L}_{W\Phi, B\Phi}$	\mathcal{L}'_W	$\mathcal{L}'_{W\Phi}$
$ff(V)V_T V_T$	$\boxed{0}$	$\boxed{2}$	$\boxed{0}$	$\boxed{2}$	$\boxed{0}$
$fV_T(f)\bar{f}V_T$	$\boxed{0}$	—	—	—	—

Table 3c: Same as Table 3a for $f\bar{f} \rightarrow V_T V_T$.